

Zadania z analizy matematycznej; Zestaw 2 30.10.2010

Proszę znaleźć granice ciągów:

$$1. \lim_{n \rightarrow \infty} \frac{3n^3 + 17n - 4}{2n^3 - 49n^2 + 117}$$

$$2. \lim_{n \rightarrow \infty} \frac{2n^3 - 14n + 99}{7n^2 + 14n + 17}$$

$$3. \lim_{n \rightarrow \infty} \frac{13n^2 - 5n + 11}{5n^3 - 114n - 13}$$

$$4. \lim_{n \rightarrow \infty} \frac{4n^3 + 5n^4 + 16}{2n^2 - 3n^4 - 16}$$

$$5. \lim_{n \rightarrow \infty} \frac{5n^4 - 3n^5 + 12n^2}{2n^3 + n^5 - 11}$$

$$6. \lim_{n \rightarrow \infty} \frac{4 + 5n + 7n^2}{3 + 2n^2 + n^3}$$

$$7. \lim_{n \rightarrow \infty} \frac{(2n+3)^2(3n-4)^3}{(2n-1)^5}$$

$$8. \lim_{n \rightarrow \infty} \frac{(2n-1)^3}{(n+3)(3n-1)}$$

$$9. \lim_{n \rightarrow \infty} \sqrt{n^2 + n + 1} - \sqrt{n^2}$$

$$10. \lim_{n \rightarrow \infty} \sqrt{n^2 + 3n - 2} - \sqrt{n^2 + n + 3}$$

$$11. \lim_{n \rightarrow \infty} \sqrt{n^2 + 2n + 7} - \sqrt{n^2 - n + 1}$$

$$12. \lim_{n \rightarrow \infty} \frac{\sin n}{n+3}$$

$$13. \lim_{n \rightarrow \infty} \frac{\cos^2 n + 3}{n^2 - 1}$$

$$14. \lim_{n \rightarrow \infty} \frac{3 + 4^n}{2^{2n} - 2}$$

$$15. \lim_{n \rightarrow \infty} \frac{9^n + 3^n + 7}{3^{2n} + 3}$$

$$16. \lim_{n \rightarrow \infty} \sqrt[n]{3^n + 2^n}$$

$$17. \lim_{n \rightarrow \infty} \sqrt[n]{7^n - 3^n}$$

$$18. \lim_{n \rightarrow \infty} \sqrt[n]{3 \cdot 6^n + 2^n}$$

$$19. \lim_{n \rightarrow \infty} \left(1 + \frac{3}{n}\right)^n$$

$$21. \lim_{n \rightarrow \infty} \left(\frac{1+n}{n+2}\right)^n$$

$$22. \lim_{n \rightarrow \infty} \left(\frac{2n-3}{2n+1}\right)^n$$

$$23. \lim_{n \rightarrow \infty} \left(\frac{n^2 - n + 1}{n^2}\right)^{n^2}$$

$$24. \lim_{n \rightarrow \infty} \left(\frac{n+3}{n+7}\right)^{n+4}$$

Proszę znaleźć granice funkcji (uwaga na punkt, w którym liczona jest granica):

$$25. \lim_{x \rightarrow \infty} \frac{3x^3 + x^2 - 13}{2x^3 + 3x + 7}$$

$$26. \lim_{x \rightarrow 1} \frac{5x+1}{3x-1}$$

$$27. \lim_{x \rightarrow 1} \frac{3x^2 - 5x + 2}{x^2 + 3x - 2}$$

$$28. \lim_{x \rightarrow 2} \frac{3x^2 - 5x - 2}{x^2 - 3x + 2}$$

$$29. \lim_{x \rightarrow 1} \frac{2x^2 + x - 3}{5x^2 - 3x - 2}$$

$$30. \lim_{x \rightarrow 0} \frac{\sin 3x}{2x}$$

$$31. \lim_{x \rightarrow 0} \frac{\sin 7x}{\sin 5x}$$

$$32. \lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 1} - \sqrt{x + 1}}{x}$$

$$33. \lim_{x \rightarrow 1} \frac{\sqrt{x^2 + 1} - \sqrt{x + 1}}{x - 1}$$

$$34. \lim_{x \rightarrow \pi} \frac{1 + \cos x}{\sin^2 x}$$

$$35. \lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{3x}$$

$$36. \lim_{x \rightarrow 0} \frac{x}{\operatorname{arctg} x}$$

$$37. \lim_{x \rightarrow \infty} \left(\frac{3+x}{x+2}\right)^x$$

$$38. \lim_{x \rightarrow 0} (1 - 2x)^{\frac{1}{x}}$$

Proszę zbadać ciągłość następujących funkcji:

$$39. f(x) = \frac{1}{x} \text{ dla } x \neq 0$$

$$40. f(x) = \frac{1}{x} \text{ dla } x \neq 0, f(0) = 0$$

$$41. f(x) = \frac{\sin x}{x} \text{ dla } x \neq 0, f(0) = 1$$

$$42. f(x) = \frac{x-1}{|x-1|} \text{ dla } x \neq 1, f(1) = 0$$

$$43. f(x) = |x-1| \text{ dla } x \neq 0, f(0) = 1$$